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# The Macro Model of the Inequality Process and The Surging Relative Frequency of Large Wage Incomes

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## 1 The Surge In Wage Income Nouveaux Riches in the U.S., 1961-2003

Angle (2006) shows that the macro model of the Inequality Process provides a parsimonious fit to the U.S. wage income distribution conditioned on education, 1961-2001. Such a model should also account for all time-series of scalar statistics of annual wage income. The present paper examines one such time-series, the relative frequency of large wage incomes 1961-2003. Figure 1 shows an aspect of this kind of statistic: the larger the wage income, the greater the proportional increase in its relative frequency. The phenomenon that figure 1 shows, a surge in wage income nouveaux riches, has caused some alarm and given rise to fanciful theories. The present paper shows that the macro model of the Inequality Process (IP) accounts for this phenomenon. In fact, it is simply an aspect of the way wage income distributions change when their mean and all their percentiles increase, which they do simultaneously, i.e., it is good news for a much larger population than the nouveaux riches alone. Nevertheless, many economists and sociologists have interpreted the surge in wage income nouveaux riches<sup>1</sup> as an alarming bifurcation of the U.S. wage income distribution into two distributions, one poor, the other rich, a

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<sup>1</sup> The term nouveaux riches perhaps brings to mind the new wealth of entrepreneurs most of whose income is from tangible assets. Nouveaux riches is used here only to name the earners of wage income who have begun to earn a wage income much larger than the average.

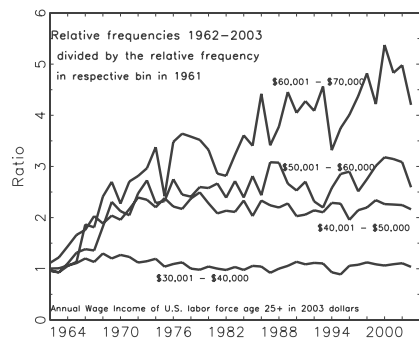


Fig. 1

**Fig. 1.** Ratio of relative frequencies 1962 – 2003 in wage income bins \$1-\$10,000, \$10,001-\$20,000 etc. in terms of constant 2003 dollars to the relative frequency in each bin in 1961 Source: Author’s estimates of data of the March Current Population Survey.



Fig. 2

**Fig. 2.** Source: Author’s estimates from March CPS data.

‘hollowed out’ distribution. Fear of the ‘hollowing out’ of the U.S. wage income distribution has not only roiled academia but has resulted in alarmed press reports and even become an issue in the 2004 U.S. presidential campaign.

The present paper shows that an increase in mean wage income decreases the relative frequency of wage incomes smaller than the mean and increases the relative frequency of wage incomes greater than the mean. Distance from the mean of a particular wage income, call it  $x_0$ , is a factor in how fast the relative frequency of wage incomes of that size change. For  $x_0$ ’s greater than the mean, the greater  $x_0$ , the greater the proportional growth in its relative frequency. There is an analogous and compensating decrease in the relative frequency of wage incomes smaller than the mean. The IP’s macro model implies that the wage income distribution stretches to the right when the unconditional mean of wage income increases, explaining both the surge in wage income nouveaux riches and the fact that the bigger wage income percentile has grown more than the smaller wage income percentile. Data on U.S. wage incomes 1961-2003 confirm the implications of the IP’s macro model.

The data on which this paper is based are the pooled cross-sectional time series formed from ‘public use samples’ of the records of individual respondents to the March Current Population Surveys (CPS) 1962-2004. The March CPS is a survey of a large sample of U.S. households conducted by the U.S. Bureau of the Census.<sup>2</sup> Each March CPS asks questions about the level of education of

<sup>2</sup> These data have been cleaned, documented and made readily accessible as a user-friendly database by Unicon Research Corporation (2004), a public data reseller supported by the (U.S.) National Institutes of Health. The author welcomes repli-

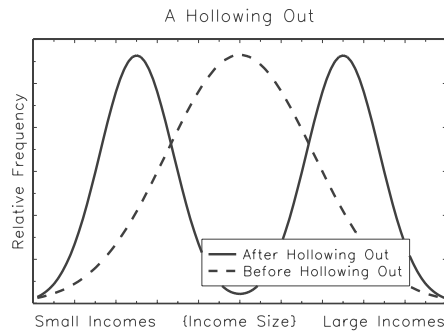


Fig. 3

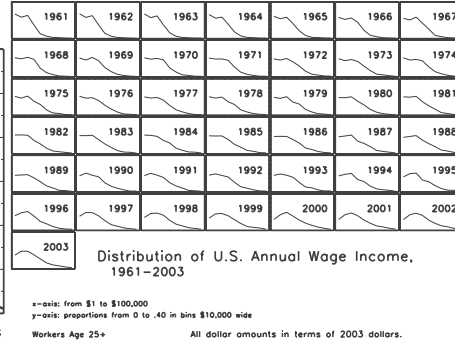


Fig. 4

**Fig. 3.** Caricature of the interpretation in the popular press of the transformation of the U.S. wage income distribution into a bimodal, ‘barbell’ distribution in recent decades. See Appendix B.

**Fig. 4.** Source: Author’s estimates from March CPS data.

members of the household and their sources of income in the previous calendar year. The study population is U.S. residents with at least \$1 (one U.S. dollar) in annual wage income who are at least 25 years old. All dollar amounts in this paper are in terms of constant 2003 dollars, i.e., adjusted for changes in the purchasing power of a dollar from 1961 through 2003. See Appendix A.

Figure 1 illustrates one of two related ways to measure the surge in wage income nouveaux riches in the U.S. 1961-2003. Figure 1 measures change in the relative frequencies in the far right tail of the wage income distribution, the distribution of wage income recipients over the largest wage incomes. This dynamic of the distribution can be readily understood in terms of the algebra of the macro model of the Inequality Process. The other way to measure the surge is via change in percentiles of wage incomes (in constant dollars), e.g., the 90th percentile. Figure 2 shows the 90th percentile of wage incomes increasing more in constant dollars than the 10th percentile, a small wage income. This second way of measuring the surge, as a stretching of the distribution to the right, is implied by the IP’s macro model but requires numerical integration to demonstrate.

The surge in wage income nouveaux riches in the U.S. 1961-2003 has been a focus of concern in U.S. labor economics and sociology journals. A substantial fraction of contributions to this literature have interpreted the surge as part of the transformation of the U.S. wage income into a bimodal, U-shaped distribution. See figure 3 for a caricature of this thesis and the ‘hollowing

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cation of tests of the Inequality Process either with the data used in this paper, available at nominal cost from the Unicon Research Corporation, 111 University Drive, East, Suite 209, College Station, Texas 77840, USA, or replication with comparable data from additional countries.

out' literature itself (Kuttner, 1983; Thurow, 1984; Lawrence, 1984; Blackburn and Bloom, 1985; Bradbury, 1986; Horrigan and Haugen, 1988; Coder, Rainwater, and Smeeding, 1989; Levy and Michel, 1991; Duncan, Smeeding and Rodgers, 1993; Morris, Bernhardt, and Handcock, 1994; Wolfson, 1994; Esteban and Ray, 1994; Jenkins, 1995; Beach, Chaykowski, and Slotsve, 1997; Wolfson, 1997; Burkhauser, Crews Cutts, Daly, and Jenkins, 1999; Esteban and Ray, 1999; Duclos, Esteban and Ray, 2004;). The emergence of a U-shaped wage income distribution has been termed the 'hollowing out' or 'polarization' of the wage income distribution. A 'hollowed out' distribution has also been called a 'barbell distribution'. In a 'hollowing out' the relative frequency of middling wage incomes decreases while the relative frequencies of small and large wage incomes increase. The 'hollowing out' thesis explains the surge in large wage incomes but it is burdened with having to hypothesize a surge in small wage incomes as well. The U.S. labor economics and sociology literatures on wage income measure trends in terms of scalar statistics of wage income, mostly the median plus the grab bag of statistics referred to under the rubric 'statistics of inequality'. Models of the dynamics of the distribution are rare and never prominent in this literature. Thus, hypothesized dynamics of the distribution have been used in this literature to explain trends in the scalar statistics without confirmation of what the empirical distribution has actually been doing. The rise of the thesis of the 'hollowing out' of the U.S. wage income distribution requires either that researchers were unaware of how the empirical distribution of wage incomes in the U.S. had changed, or, once the 'hollowing out' interpretation of how the distribution had changed had become established in the literature and popularized in the press, editors and reviewers were unable to accept evidence to the contrary, i.e., the journals that established the 'hollowing out' thesis could not correct their error.<sup>3</sup>

Figure 4 displays estimates of the U.S. wage income distribution from 1961 through 2003. It is clear that, indeed, its right tail thickened over these 43 years, i.e., the relative frequency of large wage incomes increased validating half of the 'hollowing out' hypothesis. However, it is as clear in figure 4 that the left tail of the distribution of wage incomes, the distribution of workers over small wage incomes, thinned, that is, the relative frequency of small wage incomes decreased. The 'hollowing out' thesis requires both to increase simultaneously.

Figure 5 displays the forward differences between mean relative frequencies of wage income between two ten year periods, 1961-1970 and 1994-2003. Ten year means are taken to smooth the relative frequencies. Figure 5 shows that the relative frequency of small wage incomes fell between these two periods and larger wage incomes increased, just as an inspection of figure 4 would lead one to believe.

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<sup>3</sup> The author tried to correct this literature in terms familiar to its contributors over a period of many years but, so far, has been unable to publish in any of the journals responsible for popularizing the 'hollowing out' thesis.

## 2 The Micro- and Macro-Models of the Inequality Process (IP)

The Inequality Process (IP) is a stochastic binary interacting particle system derived from an old verbal theory of economic anthropology (Angle, 1983 to 2006). People are the particles. Wealth is the positive quantity transferred between particles. The theory from which the IP is derived is the ‘surplus theory of social stratification’. It asserts that economic inequality among people originates in competition for surplus, societal net product. The IP literature dates from (Angle, 1983). The IP is an abstraction of a mathematical model from Gerhard Lenski’s (1966) speculative extension of the surplus theory to account for the decrease in wealth inequality with techno-cultural evolution. Lenski thought that more skilled workers could bargain for a larger share of what they produce. Lux (2005) introduced econophysicists to the IP at Econophys-Kolkata I and pointed out that the econophysics literature on wealth and income distribution had replicated some of Angle’s earlier Inequality Process findings.

The pair of transition equations of the Inequality Process (IP) for competition for wealth between two particles is called here the micro model of the IP to distinguish the IP’s model of particle interactions, the micro-level, from the model that approximates the solution of the micro model in terms of its parameters, the distribution of wealth, the macro model.

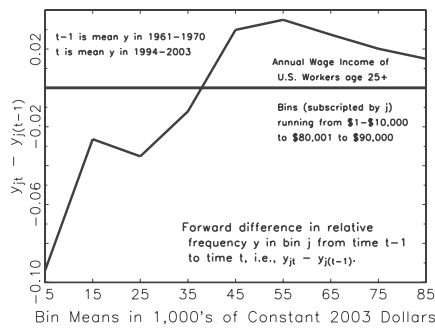


Fig. 5

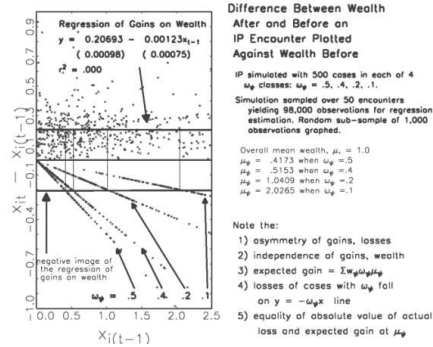


Fig. 6

**Fig. 5.** Forward difference between mean relative frequencies in 1961-1965 ( $t-1$ ) and mean relative frequencies in 1999-2003 ( $t$ ). Source: Author’s estimates from data of the March CPS.

**Fig. 6.** The scattergram of wealth changes in the population of particles from time  $t-1$  to  $t$  plotted against wealth at time  $t-1$ .

## 2.1 The Micro-Model of the Inequality Process (IP)

Since the macro-model of the Inequality Process (IP) is derived from the IP's micro model, description of the former should begin with description of the latter. Consider a population of particles in which particles have two characteristics. Wealth is one such trait. Particle  $i$ 's wealth at time  $t$  is denoted,  $x_{it}$ . Wealth is a positive quantity that particles compete for pairwise in zero-sum fashion, i.e., the sum of wealth of two particles after their encounter equals the sum before their encounter. The other particle characteristic is the proportion of wealth each loses when it loses an encounter. That proportion is the particle's parameter  $u$ . So, from the point of view of a particular particle, say particle  $i$ , the proportion of wealth it loses, if it loses,  $\omega_i$ , is predetermined. When it wins, what it wins is, from its point of view, a random amount of wealth. Thus there is an asymmetry between gain and loss from the point of view of particle  $i$ . Long term each particle wins nearly 50% of its encounters. Wealth is transferred, long term, to particles that lose less when they lose, the robust losers.

Let particle  $i$  be in the class of particles that loses an  $\omega_\psi$  fraction of their wealth when they lose, i.e.,  $\omega_i = \omega_\psi$ . In the IP's meta-theory, Lenski's speculation, workers who are more skilled retain a larger proportion of the wealth they produce. So smaller  $\omega_\psi$  in the IP's meta-theory represents the more skilled worker. Worker skill is operationalized in tests of the IP, as is usual in labor economics, by the worker's level of education, a characteristic readily measured in surveys. For tests of the IP on wage income data by level of worker education, the IP's population of particles is partitioned into equivalence classes of its particles'  $\omega_\psi$  by the corresponding level of education, so that the proportion formed by workers at the  $\psi$ th level of education of the whole labor force,  $u_\psi$ , (' $u$ ' to suggest 'weight'), is the proportion formed by the  $\omega_\psi$  equivalence class of the population of particles. The  $u_\psi$ 's are estimated from data and so are the  $\omega_\psi$ 's by fitting the comparable statistic of the IP to either micro-level data (the dynamics of individual wage incomes) or macro-level data (the dynamics of wage income distributions). The IP's meta-theory implies that estimated  $\omega_\psi$ 's should scale inversely with worker education level, based on the assumption that the more educated worker is the more productive worker. Nothing in the testing of this prediction forces this outcome. The predicted outcome holds in U.S. data on wage incomes by level of education as demonstrated in Angle (2006) for 1961-2001 and below for 1961-2003. While there is no apparent reason why this finding should not generalize to all industrial labor forces in market economies, the universality of the prediction is not yet established. The IP is a highly constrained model. Its predictions are readily falsified if not descriptive of the data.

The transition equations of the competitive encounter for wealth between two particles in the Inequality Process' (IP's) micro model are:

$$\begin{aligned} x_{it} &= x_{i(t-1)} + \omega_{\theta j} d_{it} x_{j(t-1)} - \omega_{\psi i} (1 - d_{it}) x_{i(t-1)} \\ x_{jt} &= x_{j(t-1)} - \omega_{\theta j} d_{it} x_{j(t-1)} + \omega_{\psi i} (1 - d_{it}) x_{i(t-1)} \end{aligned} \quad (1)$$

where  $x_{i(t-1)}$  is particle  $i$ 's wealth at time  $t - 1$ ,

$$d_{it} = \begin{cases} 1 & \text{with probability .5 at time } t \\ 0 & \text{otherwise.} \end{cases}$$

and,

$\omega_{\psi i}$  = proportion of wealth lost by particle  $i$  when it loses (the subscript indicates that particle  $i$  has a parameter whose value is  $\omega_{\psi i}$ ; there is no implication that the  $\omega_{\psi}$  equivalence class of particles has only one member or that necessarily  $\omega_{\psi i} \neq \omega_{\theta j}$ );

$\omega_{\theta j}$  = proportion of wealth lost by particle  $j$  when it loses.

Particles are randomly paired; a winner is chosen via a discrete 0,1 uniform random variable; the loser gives up a fixed proportion of its wealth to the winner. In words, the process is:

Randomly pair particles. One of these pairs is particle  $i$  and particle  $j$ . A fair coin is tossed and called. If particle  $i$  wins, it receives an  $\omega_{\theta}$  share of particle  $j$ 's wealth. If particle  $j$  wins, it receives an  $\omega_{\psi}$  share of particle  $i$ 's wealth. The other particle encounters are analogous. Repeat.

The asymmetry of gain and loss is apparent in figure 6, the graph of forward differences,  $x_{it} - x_{i(t-1)}$  against wealth,  $x_{i(t-1)}$ , resulting from (1). The Inequality Process differs from the Saved Wealth Model, a modification of the stochastic model of the Kinetic Theory of Gases that generates a gammoidal stationary distribution discussed by Chakraborti, Chakrabarti (2000); Chatterjee, Chakrabarti, and Manna (2003); Patriarca, Chakraborti, and Kaski (2004); Chatterjee, Chakrabarti, and Manna (2004); Chatterjee, Chakrabarti, and Stinchcombe (2005); Chatterjee, Chakraborti, and Stinchcombe (2005); Patriarca, Chakraborti, Kimmo, and Germano (2005). The following substitution converts the Inequality Process into the Saved Wealth Model (apart from the random  $\omega$  factor in Chatterjee et al, 2004 and subsequent papers):

$$d_{it} \rightarrow \epsilon_{it}$$

where  $\epsilon_{it}$  is a continuous, uniform i.i.d random variate with support at  $[0.0, 1.0]$ .

## 2.2 The Macro-Model of the Inequality Process (IP)

The macro model of the Inequality Process (IP) is a gamma probability density function (pdf),  $f_{\psi t}(x)$ , a model of the wage income,  $x$ , of workers at the same level of education, the  $\psi$ th at time  $t$ . The macro model approximates the stationary distribution of wealth of the IP's micro model. The macro model was developed in a chain of papers (Angle, 1993, 1996-2001, 2002b-2006). The IP's macro model in the  $\omega_{\psi}$  equivalence class is:

$$f_{\psi t}(x) = \frac{\lambda_{\psi t}^{\alpha_{\psi}}}{\Gamma(\alpha_{\psi})} x^{\alpha_{\psi}-1} \exp(-\lambda_{\psi t} x) \quad (2)$$

or in terms of the IP's parameter in the  $\omega_\psi$  equivalence class:

$$f_{\psi t}(x) = \exp \left[ \left( \frac{1 - \omega_\psi}{\omega_\psi} \right) \ln \left( \frac{1 - \omega_i}{\tilde{\omega}_t \mu_t} \right) \right] \times \exp \left[ -\ln \Gamma \left( \frac{1 - \omega_\psi}{\omega_\psi} \right) + \left( \frac{1 - 2\omega_i}{\omega_i} \right) \ln(x) - \left( \frac{1 - \omega_i}{\tilde{\omega}_t \mu_t} \right) x \right] \quad (3)$$

where:

$\alpha_\psi \equiv$  the shape parameter of the gamma pdf that approximates the distribution of wealth,  $x$ , in the  $\omega_\psi$  equivalence class, intended to model the wage income distribution of workers at the  $\psi$ th level of education regardless of time;  $\alpha_\psi > 0$

$$\alpha_\psi \approx \frac{1 - \omega_\psi}{\omega_\psi} \quad (4)$$

and:

$\lambda_{\psi t} \equiv$  scale parameter of distribution of the gamma pdf that approximates the distribution of wealth,  $x$ , in the  $\omega_\psi$  equivalence class, intended to model the wage income distribution of workers at level  $\psi$  of education in a labor force with a given unconditional mean of wage income and a given harmonic mean of  $\omega_\psi$ 's at time  $t$ ;

$$\lambda_{\psi t} > 0$$

$$\lambda_{\psi t} \approx \frac{(1 - \omega_\psi) \left( \frac{u_{1t}}{\omega_1} + \dots + \frac{u_{\psi t}}{\omega_\psi} + \dots + \frac{u_{\Psi t}}{\omega_\Psi} \right)}{\mu_t} \approx \frac{(1 - \omega_\psi)}{\tilde{\omega}_t \mu_t} \quad (5)$$

where:

$\mu_t =$  unconditional mean of wage income at time  $t$

$\tilde{\omega}_t =$  harmonic mean of the  $\omega_\psi$ 's at time  $t$ .

and  $\mu_t$  and the  $u_{\psi t}$ 's are exogenous and the sole source of change in a population of particles where  $\Psi$   $\omega$  equivalence classes are distinguished. Consequently, the dynamics of (2), the IP's macro model, are exogenous, that is, driven by the product  $(\tilde{\omega}_t \mu_t)$  and expressed as a scale transformation, i.e., via  $\lambda_{\psi t}$ . Figure 7 shows the shapes of a gamma probability density function (pdf) for a fixed scale parameter and several values of the shape parameter,  $\alpha_\psi$ . Figure 7 shows that if the IP's meta-theory is correct, more education, operationalized as smaller  $\omega_\psi$  earns a worker a place in a wage income distribution with a larger  $\alpha_\psi$ , a more centralized distribution, whose mean, equal to  $\alpha_\psi / \lambda_{\psi t}$ , is larger than that of the worker with less education.

Comparison of figures 7, 8, and 9 show the consequences of change in the gamma scale parameter on a gamma distribution holding the shape parameters constant. A decrease in  $\lambda_\psi$  stretches the mass of the pdf to the right over larger  $x$ 's as in figure 8, increasing all percentiles of  $x$ . Compare figure 8 to figure 7. In the IP, particles circulate randomly within the distribution of wealth of their  $\omega_\psi$  equivalence class, so an increase in  $\mu_t$  of a magnitude sufficient to increase the product  $(\tilde{\omega}_t \mu_t)$  and decrease  $\lambda_{\psi t}$  may not mean that each



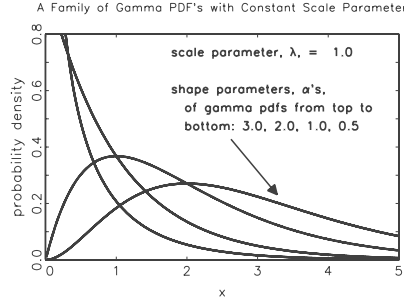


Fig. 7

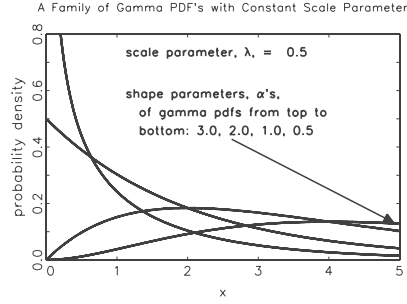


Fig. 8

**Fig. 7.** A family of gamma pdfs with different shape parameters but the same scale parameter, 1.0.

**Fig. 8.** A family of gamma pdfs with different shape parameters but the same scale parameter, 0.5.

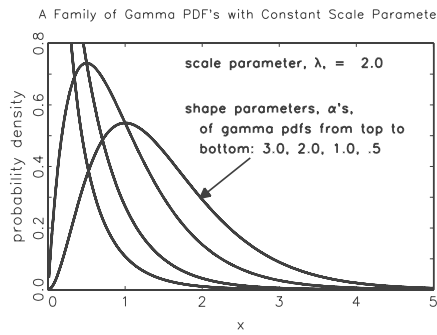


Fig. 9

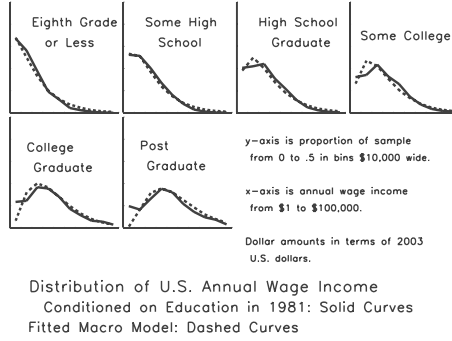


Fig. 10

**Fig. 9.** A family of gamma pdfs with different shape parameters but the same scale parameter, 2.0.

**Fig. 10.** Source: author's estimates based on March, CPS data.

and every particle in the  $\omega_\psi$  equivalence class increases its wealth, although all the percentiles increase.  $\tilde{\omega}_t$  is expected to decrease given a rising level of education in the U.S. labor force.  $\mu_t$ , the unconditional mean of wage income, rose irregularly in the U.S. in the last four decades of the 20th century.

If proportional increase in  $\mu_t$  offsets proportional decrease in  $\tilde{\omega}_t$  then the product  $(\tilde{\omega}_t \mu_t)$  increases,  $\lambda_{\psi t}$  decreases, and the IP's macro model implies that wage income distribution is stretched to the right as in figure 8 with all percentiles of wage income increasing. However, if the product  $(\tilde{\omega}_t \mu_t)$  decreases, then  $\lambda_{\psi t}$  increases and the IP's macro model predicts that the wage income distribution is compressed to the left, that is, its mass is moved over

smaller wage income amounts and its percentiles decrease as in figure 9 by comparison to figures 7 and 8.

The product  $(\tilde{\omega}_t \mu_t)$  is estimated in the fit of the IP's macro model to the 43 distributions of wage income conditioned on education in the U.S. according to the March CPS' of 1962 through 2004, which collected data on wage incomes in 1961 through 2003. Six levels of worker education level have been distinguished. See Table 1. There are  $43 \times 6 = 258$  partial distributions to be fitted. Also fitted are 258 median wage incomes, one for each partial distribution fitted. See Appendix B. Each partial distribution has fifteen relative frequency bins, each \$10,000 (in constant 2003 dollars) wide, e.g., \$1 - \$10,000, \$10,001-\$20,000, etc, for a total of  $258 \times 15 = 3,870$   $x$  (income),  $y$  (relative frequency) pairs to be fitted by the IP's macro model which has six degrees of freedom, the six values of  $\omega_\psi$  estimated. The fits are simultaneous. The fitting criterion is the minimization of weighted squared error, i.e., nonlinear least squares. The weight on each partial distribution in each year in the fit is  $u_{\psi t}$ , the proportion of the labor force with  $\psi$ th level of education in that year. A search is conducted over the parameter vector via a stochastic search algorithm that is a variant of simulated annealing to find the six values that minimize squared error. The squared correlation between the 3,870 observed and expected relative frequencies is .917. Table 1 displays the estimated parameters and their bootstrapped standard errors. Note that the estimated  $\omega_\psi$ 's scale inversely with level of education as predicted by the IP's meta-theory. Figure 10 displays the IP macro model's fit to the six partial distributions of wage income by level of education in 1981.

**Table 1.** Estimates of the Parameters of the IP's Macro-Model

Highest Level of Education	$\omega_\psi$ estimated by fitting the macro-model to 258 partial distributions (43 years X 6 levels of education)	bootstrapped standard error of $\omega_\psi$ (100 re-samples)	estimate of $\alpha_\psi$ corresponding to $\omega_\psi$
eighth grade or less	0.4524	.0009582	1.1776
some high school	0.4030	.0006159	1.4544
high school graduate	0.3573	.0004075	1.7924
some college	0.3256	.0005033	2.0619
college graduate	0.2542	.0007031	2.7951
post graduate education	0.2084	.0005216	3.6318

Separately, 258 gamma pdfs, each with two unconstrained parameters, were also fitted to each of the 258 partial distributions, a 516 parameter fit. These fits were done to create an alternative model to baseline how much less well the IP's macro model did than unconstrained gamma pdf fits to the same data set. The squared correlation between the 3,870 observed and expected relative frequencies under this alternative model is .957. Thus the IP's macro

model fits the data almost as well as the unconstrained gamma pdf alternative model although the IP's macro model uses only 6 degrees of freedom and the alternative model 516.

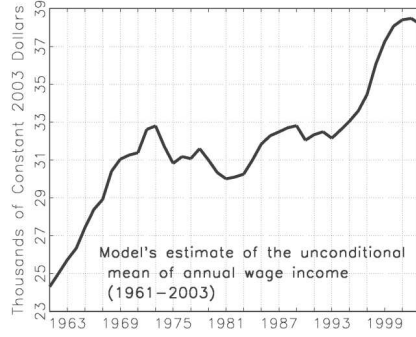


Fig. 11

**Fig. 11.** Source: Author's estimates from data of the March Current Population Survey.

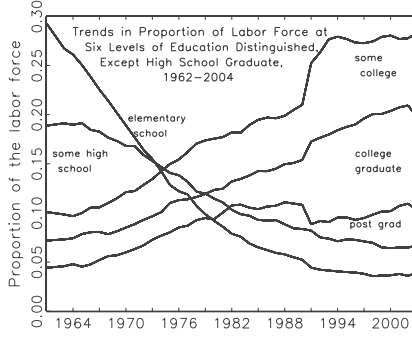


Fig. 12

**Fig. 12.** Source: Author's estimates from data of the March Current Population Survey.

Figure 11 shows that the unconditional mean of wage income in the U.S. increased substantially in the 1960's and again in the 1990's in constant 2003 dollars. There was a smaller move upward in the early to mid 1980's. However between the early 1970's and mid-1990's there were small declines and small increases that netted each other out, i.e., the unconditional mean of wage income in the U.S. did not increase in constant dollar terms for over two decades. The IP's macro model implies that the scale factor of wage income at each level of education in the labor force is driven by the product  $(\tilde{\omega}_t \mu_t)$ . In the model bigger  $(\tilde{\omega}_t \mu_t)$  stretches the distribution of wage incomes at each level of education to the right over larger wage incomes.  $\mu_t$  has to increase proportionally more than  $\tilde{\omega}_t$  decreases for all percentiles of the distribution conditioned on education to increase.

$\tilde{\omega}_t$  is the harmonic mean of the  $\omega_\psi$ 's at each time point. The proportion each  $\omega_\psi$  equivalence class forms of the population,  $u_{\psi t}$ , changes as the proportion of workers at a given educational level changes in the labor force. From 1961-2003 the level of education of the U.S. labor force rose substantially. See figure 12. Given the  $\omega_\psi$ 's estimates in Table 1,  $\tilde{\omega}_t$  decreases 1961-2003 as the level of education rises in the U.S. labor force. Figure 13 displays the course of  $\tilde{\omega}_t$  from 1961 through 2003, a steady decline throughout.

Figure 14 graphs the estimated product  $(\tilde{\omega}_t \mu_t)$  over time. Note that decline between the early 1970's and mid-1990's was much larger proportionally than in figure 11, the time-series of  $\mu_t$ . That means that the U.S. wage in-

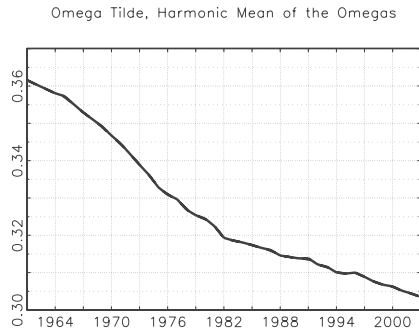


Fig. 13

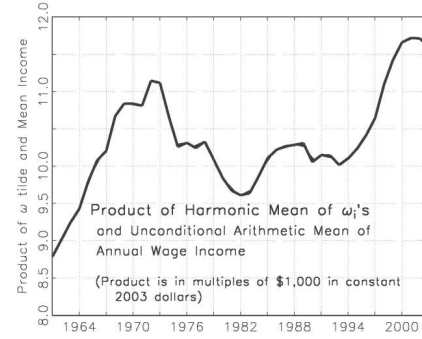


Fig. 14

**Fig. 13.** Source: Author's estimates from data of the March Current Population Survey.

**Fig. 14.** Product of the unconditional mean,  $\mu_t$ , and the harmonic mean of the  $\omega_{\psi}$ 's,  $\tilde{\omega}_t$ . Source: Author's estimates from data of the March Current Population Survey.

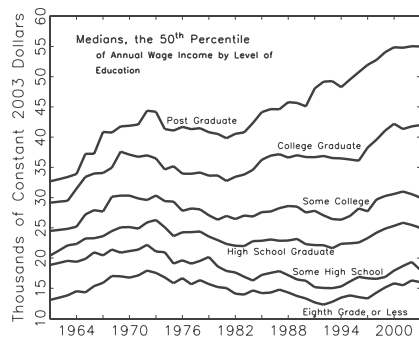


Fig. 15

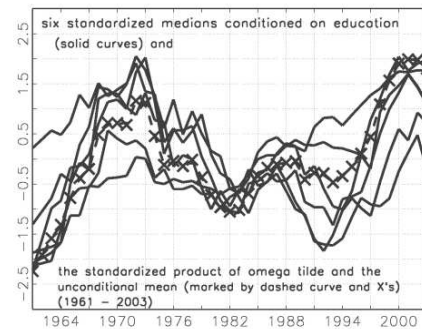


Fig. 16

**Fig. 15.** Source: Author's estimates from data of the March Current Population Survey.

**Fig. 16.** Source: Author's estimates from data of the March Current Population Survey.

come distribution conditioned on education was compressed substantially to the left over smaller wage incomes from 1976 through 1983 in a much more pronounced way than figure 11 implies. Figure 11 incorporates the positive effect on the unconditional mean of the rise in education level in the U.S. labor force from 1961 through 2003. Figure 14 shows the negative effect of the rise in educational level on wage earners, few of whom raise their education level while they work for a living. The rise in education level in the labor force as a whole is due to the net increase occasioned by the entry of more

educated younger workers and the exit of less educated older workers. Figure 14 shows that most wage earners, those not raising their education level while they worked, experienced a decrease of wage income percentiles during the 1970's and early 1980's, the sort of wage income compression toward smaller wage incomes shown in the comparison of figure 9 to figure 8.

Figure 15 confirms that smaller  $(\tilde{\omega}_t \mu_t)$  did result, as the IP's macro model implies, in downturns in the medians of wage earners at each level of education from the early 1970's through the 1990's with an exception in the high, 'open-end' category of education. Its mean level of education rose. Standardization of the time-series of  $(\tilde{\omega}_t \mu_t)$  in figure 14 and standardization of the 6 time-series of conditional median wage incomes in figure 15 allow direct observation of how closely these six are associated with  $(\tilde{\omega}_t \mu_t)$ . Figure 16 shows the graphs of the 7 standardized time-series. The time-series of the standardized  $(\tilde{\omega}_t \mu_t)$ 's is marked by  $X$ 's. The 6 standardized medians track the standardized  $(\tilde{\omega}_t \mu_t)$ 's. Table 2 shows that 4 of the 6 time-series of conditional medians are more closely correlated with the product  $(\tilde{\omega}_t \mu_t)$  than with the unconditional mean,  $\mu_t$ , alone. The IP's macro model implies a larger correlation between the time-series of median wage income conditioned on education and  $(\tilde{\omega}_t \mu_t)$  than with  $\mu_t$  alone. This inference follows from Doodson's approximation formula for the median of the gamma pdf,  $f_{\psi t}(x)$ ,  $x_{(50)\psi t}$  (Weatherburn, 1947:15 [cited in Salem and Mount, 1974: 1116]):

Mean - Mode  $\approx 3$  (Mean - Median)  
in terms of a two parameter gamma pdf:

$$x_{(50)\psi t} \approx \frac{3\alpha_{\psi} - 1}{3\lambda_{\psi t}}$$

and given (4) and (5):

$$x_{(50)\psi t} \approx \left( \frac{1 - \frac{4}{3}\omega_{\psi}}{1 - \omega_{\psi}} \right) \left( \frac{\tilde{\omega}_t \mu_t}{\omega_{\psi}} \right) \quad (6)$$

a constant function of the conditional mean,  $(\tilde{\omega}_t \mu_t)/\omega_{\psi}$ .

**Table 2.** Estimated Correlations Between Time-Series

Highest Level of Education	correlation between $\tilde{\omega}_t \mu_t$ and median wage income at a given level of education	correlation between the unconditional mean $\mu_t$ and median wage income at a given level of education
eighth grade or less	.5523	.1837
some high school	.1573	-.2874
high school graduate	.8729	.5885
some college	.9279	.7356
college graduate	.9042	.9556
post graduate education	.7776	.9575

### 3 The Dynamics of the Macro Model of the Inequality Process (IP)

The dynamics of the Inequality Process (IP)'s macro-model of the wage income distribution of workers at the same level of education are driven exogenously by change in  $(\tilde{\omega}_t \mu_t)$ :

$$\begin{aligned} \frac{\partial f_{\psi t}(x)}{\partial(\tilde{\omega}_t \mu_t)} &= f_{\psi t}(x) \lambda_{\psi t} \left( \frac{x - \mu_{\psi t}}{\tilde{\omega}_t \mu_t} \right) \\ &= f_{\psi t}(x) \frac{(1 - \omega_{\psi})}{(\tilde{\omega}_t \mu_t)^2} (x - \mu_{\psi t}) \end{aligned} \quad (7)$$

where, the conditional mean of wealth in the  $\omega_{\psi}$  equivalence class,  $\mu_t$ , is:

$$\mu_{\psi t} = \frac{\alpha_{\psi}}{\lambda_{\psi t}} \approx \frac{\tilde{\omega}_t \mu_t}{\omega_{\psi}} \quad (8)$$

In (7), as  $(\tilde{\omega}_t \mu_t)$  increases,  $f_{\psi t}(x_0)$  decreases to the left of the conditional mean,  $\mu_{\psi t}$ , i.e., for  $x_0 < \mu_{\psi t}$ .  $f_{\psi t}(x_0)$  increases to the right of the conditional mean  $\mu_{\psi t}$ , i.e., for  $x_0 > \mu_{\psi t}$ . So an increase in  $(\tilde{\omega}_t \mu_t)$  simultaneously thins the left tail of the distribution of  $x$ , wealth, in the  $\omega_{\psi}$  equivalence class and thickens the right tail. (7) implies that the probability mass in the left and right tails, defined as the probability mass over  $x_0 < \mu_{\psi t}$  and  $x_0 > \mu_{\psi t}$  respectively, must vary inversely if  $(\tilde{\omega}_t \mu_t)$  changes. Thus, the macro-model of the Inequality Process (IP) squarely contradicts the hypothesis that a wage distribution conditioned on education can become U-shaped via a simultaneous thickening of the left and right tails, what the literature on the ‘hollowing out’ of the U.S. distribution of wage incomes asserts.

Given that in the IP's macro-model all change is exogenous, due to  $(\tilde{\omega}_t \mu_t)$ , the forward difference,  $f_{\psi t}(x_0) - f_{\psi(t-1)}(x_0)$ , at a given  $x_0$  can be approximated via Newton's approximation as:

$$\begin{aligned} f_{\psi t}(x_0) - f_{\psi(t-1)}(x_0) &\approx f_{\psi(t-1)}(x_0) + f'_{\psi(t-1)}(x_0) ((\tilde{\omega}_t \mu_t) - (\tilde{\omega}_{t-1} \mu_{t-1})) \\ &\quad - f_{\psi(t-1)}(x_0) \\ &\approx f_{\psi(t-1)}(x_0) \cdot \frac{(1 - \omega_{\psi})}{(\tilde{\omega}_{t-1} \mu_{t-1})} \cdot (x_0 - \mu_{\psi(t-1)}) \\ &\quad \times \left( \frac{\tilde{\omega}_t \mu_t}{\tilde{\omega}_{t-1} \mu_{t-1}} - 1 \right) \\ &\approx f_{\psi(t-1)}(x_0) \cdot \lambda_{\psi(t-1)} \cdot (x_0 - \mu_{\psi(t-1)}) \cdot \left( \frac{\tilde{\omega}_t \mu_t}{\tilde{\omega}_{t-1} \mu_{t-1}} - 1 \right) \end{aligned} \quad (9)$$

(9) says that the forward difference,  $f_{\psi t}(x_0) - f_{\psi(t-1)}(x_0)$ , of relative frequencies of the same  $x_0$  is proportional to  $f_{\psi(t-1)}(x_0)$ , the scale parameter at time  $t - 1$ ,  $\lambda_{\psi(t-1)}$ , the signed difference between  $x_0$  and the conditional

mean,  $\mu_{\psi(t-1)}$  at time  $t-1$ , and the signed proportional increase (positive) or proportional decrease (negative) in the product  $(\tilde{\omega}_{t-1}\mu_{t-1})$ . (9) implies little change in the relative frequency of wage income in the vicinity of the conditional mean. It also implies that the  $(x_0 - \mu_{\psi(t-1)})$  term can become largest in absolute value in the extreme right tail, i.e., for the largest  $x_0$ , since the absolute value of the difference  $(x_0 - \mu_{\psi(t-1)})$  is greater for the maximum  $x_0$ , typically more than three times the mean, than it is for the minimum  $x_0$ , which is very nearly one mean away from the mean. However, the forward difference,  $(f_{\psi t}(x_0) - f_{\psi(t-1)}(x_0))$ , will still be forced down toward zero in the far right tail when  $(\tilde{\omega}_{t-1}\mu_{t-1})$  increases because the RHS of (9) is multiplied by  $(f_{\psi(t-1)}(x_0))$  which becomes small quickly as  $x_0$  becomes large. So the forward difference becomes small in the far right tail even when  $(\tilde{\omega}_{t-1}\mu_{t-1})$  increases despite the fact that  $(x_0 - \mu_{\psi(t-1)})$  reaches its positive maximum for the maximum  $x_0$ . However, Newton's approximation to the ratio,  $(f_{\psi t}(x_0)/f_{\psi(t-1)}(x_0))$ , reflects the full effect of  $(x_0 - \mu_{\psi(t-1)})$  on growth in the density of the far right tail when  $(\tilde{\omega}_{t-1}\mu_{t-1})$  increases.

Given that in the IP's macro-model change is exogenous, due to  $(\tilde{\omega}_t\mu_t)$ , the ratio,  $f_{\psi t}(x_0)/f_{\psi(t-1)}(x_0)$ , is approximated via Newton's approximation as:

$$\begin{aligned} \frac{f_{\psi t}(x_0)}{f_{\psi(t-1)}(x_0)} &\approx \frac{f_{\psi(t-1)}(x_0) + f'_{\psi(t-1)}(x_0)((\tilde{\omega}_t\mu_t) - (\tilde{\omega}_{t-1}\mu_{t-1}))}{f_{\psi(t-1)}(x_0)} \\ &\approx \left[ 1 + \left[ (x_0 - \mu_{\psi(t-1)}) \left( \frac{1 - \omega_{\psi}}{\tilde{\omega}_{t-1}\mu_{t-1}} \right) \left( \frac{\tilde{\omega}_t\mu_t}{\tilde{\omega}_{t-1}\mu_{t-1}} - 1 \right) \right] \right] \end{aligned} \quad (10)$$

The bigger the  $(x_0 - \mu_{\psi(t-1)})$  term is in the right tail, the greater is the ratio  $f_{\psi t}(x_0)/f_{\psi(t-1)}(x_0)$  when  $(\tilde{\omega}_t\mu_t)$  increases. Figure 1, showing the surge in wage income nouveaux riches, graphs the empirical analogue of the ratio  $f_{\psi t}(x_0)/f_{\psi(t-1)}(x_0)$ . (10) is descriptive of figure 1. Note that according to (10), in the right tail where  $x_0 > \mu_{\psi(t-1)}$ , the difference  $(x_0 - \mu_{\psi(t-1)})$  for  $x_0$  fixed becomes smaller as the conditional mean,  $\mu_{\psi(t-1)}$ , increases with increasing  $(\tilde{\omega}_t\mu_t)$ , implying a deceleration in the rate of increase of the ratio  $f_{\psi t}(x_0)/f_{\psi(t-1)}(x_0)$  for a given increase in  $(\tilde{\omega}_t\mu_t)$ . This deceleration is evident in figure 1.

The expression for forward proportional change is that of the RHS of (10) minus 1.0:

$$\begin{aligned} \frac{f_{\psi t}(x_0) - f_{\psi(t-1)}(x_0)}{f_{\psi(t-1)}(x_0)} &\approx \frac{f'_{\psi(t-1)}(x_0)((\tilde{\omega}_t\mu_t) - (\tilde{\omega}_{t-1}\mu_{t-1}))}{f_{\psi(t-1)}(x_0)} \\ &\approx \left[ (x_0 - \mu_{\psi(t-1)}) \left( \frac{1 - \omega_{\psi}}{\tilde{\omega}_{t-1}\mu_{t-1}} \right) \left( \frac{\tilde{\omega}_t\mu_t}{\tilde{\omega}_{t-1}\mu_{t-1}} - 1 \right) \right] \\ &\approx \lambda_{\psi(t-1)}(x_0 - \mu_{\psi(t-1)}) \left( \frac{\tilde{\omega}_t\mu_t}{\tilde{\omega}_{t-1}\mu_{t-1}} - 1 \right) \end{aligned} \quad (11)$$

and it has like (9) the property that it changes sign according to whether  $x_0$  is greater than or less than the conditional mean,  $\mu_{\psi t}$ , and whether  $\tilde{\omega}_t\mu_t$

has increased or decreased. For example, in the right tail of the distribution, i.e.,  $x_0 > \mu_{\psi t}$ , when  $\tilde{\omega}_t \mu_t$  increases, forward proportional change in the distribution,  $f_{\psi t}(x_0)$ , is positive. Forward proportional change in the distribution is a product of the three factors on the RHS of (11). Forward proportional change in the distribution,  $f_{\psi t}(x_0)$ , in (11) is a linear function of the difference  $(x_0 - \mu_{\psi(t-1)})$  and can, since maximum  $x_0$  can be at least three times as far from the mean as minimum  $x_0$ , forward proportional growth in the extreme right of the right tail when  $\tilde{\omega}_t \mu_t$  increases is greater than at any other income amount. In other words, the IP's macro model implies rapid growth in the population of wage income nouveaux riches whenever  $(\tilde{\omega}_t \mu_t)$  increases. One would expect that purveyors of goods and services priced for people with large wage incomes might see their market experiencing explosive growth whenever the product  $(\tilde{\omega}_t \mu_t)$  increases.

### 3.1 The Implied Dynamics of the IP's Macro Model for the Unconditional Distribution of Wage Income

The IP's macro model of the unconditional wage income distribution, a mixture of gamma pdf's,  $f_t(x_0)$ , is:

$$f_t(x_0) = u_{1t}f_{1t}(x_0) + \dots + u_{\psi t}f_{\psi t}(x_0) + \dots + u_{\Psi t}f_{\Psi t}(x_0) \quad (12)$$

where:

$f_{\psi t}(x_0) \equiv$  IP's macro model of distribution of wealth in the  $\omega_{\psi}$  equivalence class at time  $t$ ;

$u_{\psi t} \equiv$  proportion of particles in the  $\omega_{\psi}$  equivalence class at time  $t$ , the mixing weights.

The dynamics of (12), the unconditional relative frequency of wage income, are driven by  $(\tilde{\omega}_t \mu_t)$  as in (7) and also by the direct effect of the  $u_{\psi t}$ 's:

$$\frac{\partial f_t(x_0)}{\partial(\tilde{\omega}_t \mu_t)} = \sum_{\psi} \left( u_{\psi t} f_{\psi t}(x_0) \frac{(1 - \omega_{\psi})}{(\tilde{\omega}_t \mu_t)^2} (x_0 - \mu_{\psi t}) \right) \quad (13)$$

(12) is a gamma pdf mixture; a gamma mixture is not, in general, a gamma pdf. While (12) shares many properties of (2) in the  $\omega_{\psi}$  equivalence class, it has others as well, namely the direct effect of change in the proportions, the  $u_{\psi t}$ 's, in each  $\omega_{\psi}$  equivalence class. Figure 12 shows that the  $u_{\psi t}$ 's of larger  $\omega_{\psi}$ 's (those of the less well educated, e.g. workers without a high school diploma) decreased between 1962 and 2004 while the  $u_{\psi t}$ 's of the smaller  $\omega_{\psi}$ 's (those of the more educated, e.g., with at least some post-secondary school education) increased. This change in the  $u_{\psi t}$ 's implies that  $\tilde{\omega}_t$  decreased in this period, as figure 13 shows.

The implications for the right tail of the conditional distribution,  $f_{\psi t}(x_0)$ , in (9), (10), and (11), as the product  $(\tilde{\omega}_t \mu_t)$  increases, carry through for the dynamics of the right tail of the unconditional distribution,  $f_t(x_0)$ , for  $x_0 >$



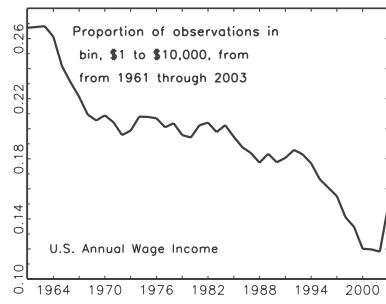


Fig. 17

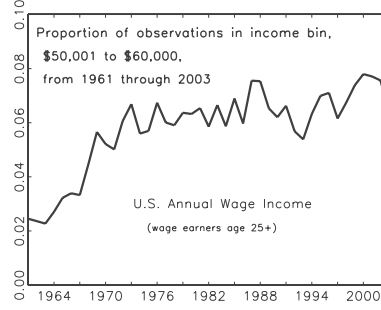


Fig. 18

**Fig. 17.** Relative frequencies of incomes \$1 - \$10,000 in the unconditional distribution. Source: Author's estimates from data of the March Current Population Survey.

**Fig. 18.** Relative frequencies of incomes \$50,001 - \$60,000 in the unconditional distribution. Source: Author's estimates from data of the March Current Population Survey.

$\mu_{\phi t}$  where  $\mu_{\phi t}$  is the mean of  $x$  in the  $\omega_{\phi}$  equivalence class where  $\omega_{\phi}$  is the minimum  $\omega$ , (and consequently  $\mu_{\phi t}$  is the maximum mean of any  $\omega$  equivalence class), and for the dynamics of the left tail of the unconditional distribution,  $f_t(x_0)$ , for  $x_0 < \mu_{\theta t}$  where  $\mu_{\theta t}$  is the mean of  $x$  in the  $\omega_{\theta}$  equivalence class, where  $\omega_{\theta}$  is the maximum  $\omega$  in the population (and consequently  $\mu_{\theta t}$  is the minimum mean of any  $\omega$  equivalence class). Thus, as  $(\tilde{\omega}_t \mu_t)$  and the  $u_{\psi}$  in equivalence classes with smaller  $\omega_{\psi}$ 's increase, (13) implies that the left tail thins and the right tail thickens. Figures 17 and 18 show that such is the case in the left tail bin, \$1-\$10,000, and the right tail bin, \$50,001 - \$60,000 (both in constant 2003 dollars). Figure 19 shows how each relative frequency (that in the bin \$1-\$10,000 and that in the bin, \$50,001-\$60,000) has a large positive correlation with other relative frequencies in the same tail and a large negative correlation with relative frequencies in the other tail. For example, the relative frequencies in the bins \$1 - \$10,000 and \$50,001 - \$60,000 have a nearly a perfect negative correlation with each other. Both relative frequencies, as one would expect given (13), have a near zero correlation with relative frequencies close to the unconditional mean of wage income.

Figure 20 shows that the unconditional forward difference of wage incomes between the average of the relative frequencies in the period 1961 to 1970 and the average of the relative frequencies in the period 1994-2003 largely overlaps the fitted forward difference between the expected relative frequencies in these two periods at the beginning and end of the time series. The time averaging is done to smooth out the pronounced frequency spiking in these data. See Angle (1994) for a discussion of frequency spiking in the wage income observations collected by the March Current Population Survey.

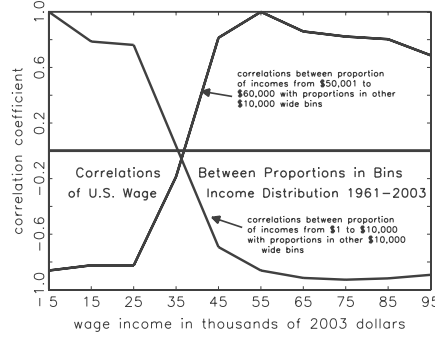


Fig. 19

**Fig. 19.** Correlations between relative frequency in wage income bin in left tail, \$1-\$10,000, and relative frequency in right tail bin, \$50,001-\$60,000, and relative frequencies in all the other income bins around the distribution. Source: Author's estimates from data of the March Current Population Survey.

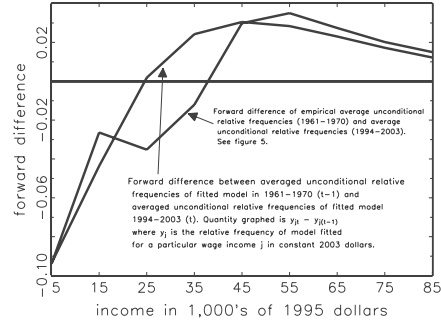


Fig. 20

**Fig. 20.** Source: Author's estimates from data of the March Current Population Survey.

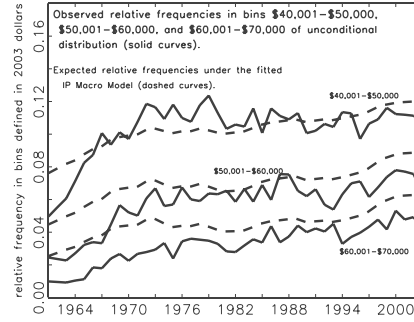
## 4 Conclusions

The IP's macro model fits the distribution of U.S. wage income conditioned on education 1961-2003. It also accounts for one of the quirkiest time-series of scalar statistics of U.S. wage income in the same time period: the more rapid growth in the relative frequency of the larger wage income in the right tail of the distribution, that is, among wage incomes greater than mean wage income. Figure 20 shows that the IP's macro model accounts for how the relative frequencies of wage income changed between 1961-1970 and 1994-2003. Figure 21 shows why, in particular, for large wage incomes (defined in constant 2003 dollars): the expected frequencies of large wage incomes under the IP's macro model track the observed frequencies of large wage incomes closely.

The observed relative frequencies are estimated from reports of personal annual wage income in the micro-data file, the individual person records, of the March Current Population Survey (CPS), in 'public use' form, i.e., with personal identifiers stripped from the file. The March CPS is a survey of a large sample of households in the U.S. conducted by the U.S. Bureau of the Census. In the March CPS, a respondent answers questions posed by a Census Bureau interviewer about members of the household. There is a question about the annual wage income of each member of the household in the previous year. See figure 4 for estimates of the distribution of annual wage income 1961-2003. All dollar amounts have been converted to constant 2003 dollars.

The U.S. Census Bureau has evaluated the adequacy of its wage income question in the March Current Population Survey (CPS) and acknowledged

that respondents, on average, underestimate the wage income they report (Roemer, 2000: 1). Roemer writes “Many people are reluctant to reveal their incomes to survey researchers and this reluctance makes such surveys particularly prone to response errors.” Roemer (2000: 17-21) reports that these underestimates are least biased downward for wage incomes near the median but seriously biased downward for large wage incomes. So it is not a problem for the IP’s macro model if it overestimates the relative frequency of large wage incomes slightly, particularly very large wage incomes, as you can see it does in figure 21.



**Fig. 21.** Unconditional right tail relative frequencies 1961-2003 (solid curves) and estimated right tail relative frequencies under the fitted IP Macro Model (dashed curves). Source: Author’s estimates from data of the March Current Population Survey.

The macro model of the Inequality Process is a gamma probability density function (pdf) whose parameters are derived from the micro model of the Inequality Process and expressed in terms of its parameters. See (2) through (5). The dynamics of this model are expressed in terms of the gamma scale parameter, (5), of this model. (5) says that the model is driven exogenously by the product  $(\tilde{\omega}_t \mu_t)$  through its scale parameter,  $\lambda_{\psi t}$ .  $(\tilde{\omega}_t \mu_t)$  is a function of the distribution of education in the labor force at time  $t$  and the unconditional mean of wage income,  $\mu_t$ , at time  $t$ .  $\tilde{\omega}_t$  is the harmonic mean of the estimated IP parameters, the  $\omega_{\psi}$ ’s. These are estimated in the fitting of the IP’s macro model to the distribution of wage income conditioned on education, 1961-2003. The  $\omega_{\psi}$ ’s also enter the formula by which the unconditional mean,  $\mu_t$ , is estimated from sample conditional medians under the hypothesis that wage income is gamma distributed. The  $u_{\psi}$ ’s, the proportions in each  $\omega_{\psi}$  equivalence class, by hypothesis the fraction of the labor force at a particular level of education, also enter the formula by which  $\mu_t$  is estimated from sample conditional medians. The IP’s macro model fits the distribution of wage income in the U.S., 1961-2003, well.

#### 4.1 The Dynamics of the Wage Income Distribution When $(\tilde{\omega}_t\mu_t)$ Increases: A Stretching, Not a ‘Hollowing Out’

##### Not a ‘Hollowing Out’

When  $(\tilde{\omega}_t\mu_t)$  increases, the distribution of wage income stretches to the right over larger wage incomes, as in the comparison of figure 8 to figure 7. Figure 8 is the graph of gamma pdf’s with different shapes but the same scale parameter. Figure 8 has gamma pdfs with the same shape parameters but a different scale parameter, one that is half that of figure 7. The gamma pdf’s of figure 8 look stretched to the right. When  $(\tilde{\omega}_t\mu_t)$  decreases, the wage income distribution is compressed to the left over smaller wage incomes, as in the comparison of figure 9 to figure 7. These effects are deduced from the IP’s macro model in (9). The last term in the product on the RHS of (9) is positive when  $(\tilde{\omega}_t\mu_t) > (\tilde{\omega}_{t-1}\mu_{t-1})$ , negative when  $(\tilde{\omega}_t\mu_t) < (\tilde{\omega}_{t-1}\mu_{t-1})$ , meaning that when  $(\tilde{\omega}_t\mu_t)$  increases, the right tail thickens, the left tail thins, and vice versa when  $(\tilde{\omega}_t\mu_t)$  decreases. While the IP’s micro model is time-reversal asymmetric, its macro model is time-reversal symmetric. The IP’s macro model implies in (10), and (11) that growth in the relative frequency of large wage incomes, i.e., the thickness of the right tail of the wage income distribution, is greater, the larger the wage income, i.e., the farther to the right in the tail, when  $(\tilde{\omega}_t\mu_t)$  increases.

So the IP’s macro model accounts for the surge in the far right tail of the wage income distribution in the U.S., the appearance of wage income nouveaux riches, as  $(\tilde{\omega}_t\mu_t)$  increased from 1961 through 2003. See figures 1, 5, 20, and 21. The IP’s macro model implies that the right tail of the wage income distribution thickened as  $(\tilde{\omega}_t\mu_t)$  increased from 1961 through 2003 and the left tail of the distribution thinned. The empirical evidence bears out this implication of the IP’s macro model, but contradicts the interpretation in the labor economics literature that the thickening of the right tail of the wage income distribution represented a ‘hollowing out’ of the wage income distribution, that is, a simultaneous thickening in the left and right tails of the distribution at the expense of the relative frequency of wage incomes near the median of the distribution, as illustrated conceptually in figure 3.

As you can see in figure 4, the unconditional distribution of wage income thinned in its left tail and thickened in its right from 1961 through 2003. Figure 17 shows how the relative frequency of wage incomes from \$1- \$10,000 (constant 2003 dollars) decreased from 1961 through 2003, although not monotonically, while figure 18 shows how the relative frequency of wage incomes from \$50,001 - \$60,000 (constant 2003 dollars) increased from 1961 through 2003, although not monotonically. \$50,001 in 2003 dollars is greater than the unconditional mean of wage income from 1961 through 2003, so the wage income bin \$50,001-\$60,000 was in the right tail the entire time. If there is any remaining question of what was happening elsewhere in the distribution, it is answered by figure 19 which shows the correlation between the relative

frequency in income bin \$1-\$10,000 with that of every other income bin. The relative frequency of this extreme left tail bin was positively correlated with the relative frequency in the other left tail bin, had almost no correlation with relative frequency of mean wage income, and a large negative correlation with relative frequencies of all the right tail income bins. Figure 19 also shows the correlations of the relative frequency of the income bin \$50,001 - \$60,000 with relative frequencies in other bins around the distribution. These correlations are a near mirror image of the correlations of the left tail bin \$1 - \$10,000. The relative frequency of income bin \$50,001 - \$60,000 has a high positive correlation with the relative frequencies of other right tail income bins, near zero correlation with the relative frequency of mean income, and a large negative correlation with the relative frequencies of left tail wage income bins. Figure 20 shows that the relative frequency of wage incomes smaller than the mean decreased between 1961 and 2003 while those greater than the mean increased. There is no doubt that the relative frequencies of the left tail of the wage income distribution vary inversely with the relative frequencies of the right tail, just as the IP's macro model implies, in contradiction of the 'hollowing out' hypothesis.

### A Stretching of the Distribution When $(\tilde{\omega}_t\mu_t)$ Increases

This paper has focused on how the relative frequency of a wage income of a given size changes when  $(\tilde{\omega}_t\mu_t)$  increases because it is algebraically transparent. The algebra indicates more rapid growth in the relative frequency of the larger wage income in the right tail of the distribution. However, a clearer demonstration of how the IP's macro model and the empirical wage income change when  $(\tilde{\omega}_t\mu_t)$  increases is in the dynamics of the percentiles of wage income, that is, not how the relative frequency of a particular fixed wage income in constant dollars,  $x_0$ , changes, but rather how the percentiles of the distribution change. Figure 2 shows that the 90th percentile of wage income increased more in absolute terms than the 10th percentile between 1961 and 2003, i.e., the distribution stretched farther to the right over larger wage incomes in its right tail than its left. Does the same occur with the 10th and 90th percentiles of the IP's macro model of the unconditional distribution of wage income? This demonstration requires numerical integration and so is less transparent algebraically than inspecting the algebra of the model for the dynamics of the relative frequency of large wage incomes.

Figure 22 displays how well the percentiles of the model track the observed percentiles of wage income. The tendency to slightly overestimate the 90th percentile is not a problem given Roemer's (2000) evaluation of the accuracy of reporting of wage income data in the March CPS. In figure 22 the graphs of the unconditional percentiles of the IP's macro model and of empirical wage income as  $(\tilde{\omega}_t\mu_t)$  increases show both distributions stretching to the right: the bigger the percentile, the more it increases in absolute constant dollars, what one would expect from the multiplication of all wage income percentiles by the

same multiplicative constant, usually greater than 1.0, in each year between 1961 and 2006.

A percentile,  $x_{(i)\psi t}$ , of the IP's macro model,  $f_{\psi t}(x)$ , is:

$$\frac{i}{100} = \int_0^{x_{(i)\psi t}} \frac{\lambda_{\psi t}^{\alpha_{\psi}}}{\Gamma(\alpha_{\psi})} x^{\alpha_{\psi}-1} \exp(-\lambda_{\psi t} x) dx$$

where  $i$  is integer and  $i$  is less than or equal to 100. Figure 15 graphs the conditional medians, the 50th percentiles,  $x_{(50)\psi t}$ 's, from 1961 through 2003. Figure 16 shows that, when standardized, i.e., when their mean is subtracted from them and this difference is divided by their standard deviation, the transformed conditional medians have a time-series close to that of the standardization of  $(\tilde{\omega}_t \mu_t)$ . (6), Doodson's approximation to the median of a gamma pdf in terms of the IP's parameters, shows why: the median is approximately a constant function of  $(\tilde{\omega}_t \mu_t)$ .  $(\tilde{\omega}_t \mu_t)$  enters  $f_{\psi t}(x)$  as a gamma scale parameter transformation, via  $\lambda_{\psi t}$ . A scale transformation affects all percentiles multiplicatively, as in the comparison of figure 8 to figure 7. The gamma pdfs of figure 8 have the same shape parameters as those of figure 7. The difference between the two sets of graphs is that those of figure 8 have scale parameters,  $\lambda_{\psi t}$ , that are one half those of figure 7. The gamma pdfs of figure 8 have been stretched to the right over larger  $x$ 's from where they were in figure 7. The IP's macro model implies this stretching to the right over larger wage incomes when the product  $(\tilde{\omega}_t \mu_t)$  increases, which figure 14 shows it did from 1961 through 2003, although not monotonically so. A larger  $(\tilde{\omega}_t \mu_t)$  results in a smaller gamma scale parameter,  $\lambda_{\psi t}$ , given (5).

So, the Inequality Process' (IP) macro model explains both the surge in the relative frequency of large wage incomes and the greater absolute increase in the greater percentile of wage incomes in the U.S., 1961-2003 as  $(\tilde{\omega}_t \mu_t)$  increased. Since the  $\tilde{\omega}_t$  term decreases with rising levels of education in the U.S. labor force, the condition of  $(\tilde{\omega}_t \mu_t)$  increasing means that the unconditional mean of wage income,  $\mu_t$ , grew more proportionally 1961-2003 than  $\tilde{\omega}_t$  decreased. Since all percentiles of wage income grew as  $(\tilde{\omega}_t \mu_t)$  increased, the surge in wage income nouveaux riches in the U.S. 1961-2003 was simply a visible indicator of generally rising wage incomes, hardly the ominous event it was made out to be by some in the scholarly literature and the popular press.

## Appendix A: The March Current Population Survey And Its Analysis

The distribution of annual wage and salary income is estimated with data from the March Current Population Surveys (CPS) (1962-2002), conducted by the U.S. Bureau of the Census. One of the money income questions asked on the March CPS is total wage and salary income received in the previous calendar year. See Weinberg, Nelson, Roemer, and Welniak (1999) for a description of the CPS and its history. The CPS has a substantial number of households in

its nationwide sample. The March Current Population Survey (CPS) provides the data for official U.S. government estimates of inequality of wage income as well as most of the labor economics literature on inequality of wage income in the U.S.

The present paper examines the civilian population of the U.S. that is 25+ in age and earns at least \$1 (nominal) in annual wage income. The age restriction to 25+ is to allow the more educated to be compared to the less educated. It is a conventional restriction in studies of the relationship of education to wage income. The data of the March CPS of 1962 through 2004 were purchased from Unicon Research, inc. (Unicon Research, inc, 2004; Current Population Surveys, March 1962-2004), which provides the services of data cleaning, documentation of variable definitions and variable comparability over time, and data extraction software. Unicon Research, inc was not able to find a copy of the March 1963 CPS public use sample containing data on education. Consequently, the distribution of wage and salary income received in 1962 (from the March 1963 CPS) conditioned on education is interpolated from the 1961 and 1963 (from the 1962 and 1964 March CPS').

All dollar amounts in the March CPS are converted to constant 2003 dollars using the U.S. Bureau of Economic Analysis National Income and Product Account Table 2.4.4 Price indexes for personal consumption expenditure by type of product [index numbers, 2000 = 100]  
<http://www.bea.gov/bea/dn/nipaweb/TableView.asp#Mid> [Last revised on 8/4/05].

## Appendix B: Estimation

### *Estimation of Relative Frequencies*

All estimates are weighted estimates. The weight associated with the  $j$ th observation in the  $t$ th year,  $u_{jt}^*$ , is:

$$u_{jt}^* = \frac{u_{jt}}{\sum_{i=1}^{n_t} u_{it}} n_t$$

where,

$u_{jt}$  = the raw weight provided by the Census Bureau for observation  $j$   
 $n_t$  = the sample size in year  $t$ .

### *Estimation of the $\mu_t$ , the Unconditional Mean, from Sample Conditional Medians, $x_{(50)\psi_t}$ 's*

While an unconditional sample mean of wage incomes in the March CPS can be directly estimated from the data, it is known to be an underestimate of the population unconditional mean,  $\mu_t$ . The sampling frame of the March CPS does not sample large wage incomes at a higher rate than smaller wage incomes. Consequently, given the right skew of the distribution wage income dollars will be missed in the form of very large individual wage incomes biasing the sample mean of wage income downward. Further, the Census Bureau itself has concluded that even when a household with one or more large

wage incomes falls into the sample, those wage income reports have a serious downward bias (Roemer, 2000:17-21). The sample median of wage incomes is robust against these problems of estimation. It is as well measured as any sample statistic of annual wage income.

The unconditional mean of the IP's macro model,  $\mu_t$ , is estimated in terms of the sample conditional medians, the  $x_{(50)\psi t}$ 's, (the median wage income at the  $\psi$ th level of education) and the  $u_{\psi t}$ 's, (the proportion of the labor force at the  $\psi$ th level of education) using Doodson's approximation formula for the median of a gamma pdf, (Weatherburn, 1947:15 [cited in Salem and Mount, 1974]) as instantiated for the IP's macro model in (6), since:

$$\mu_t = u_{1t}\mu_{1t} + u_{2t}\mu_{2t} + \dots + u_{\psi t}\mu_{\psi t} + \dots + u_{6t}\mu_{6t}.$$

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